CONCRETE
AND
CONSTRUCTIONAL ENGINEERING

INCLUDING PRESTRESSED CONCRETE

JULY 1956 **VOL. LI. NO. 7**

FIFTY-FIRST YEAR OF PUBLICATION

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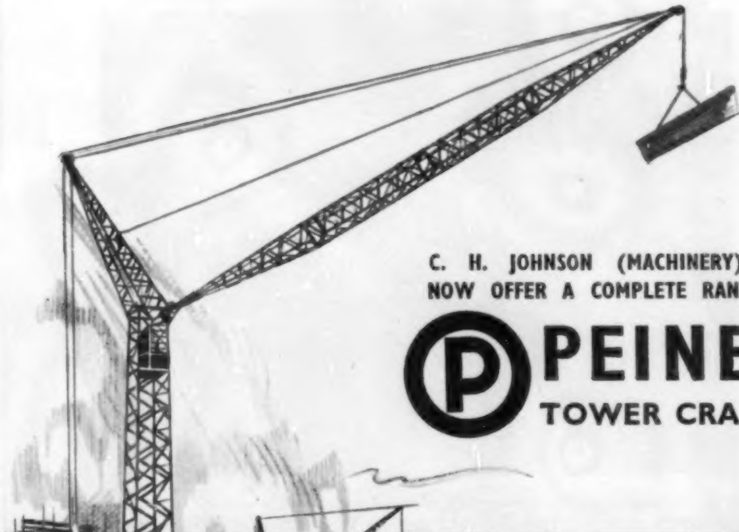


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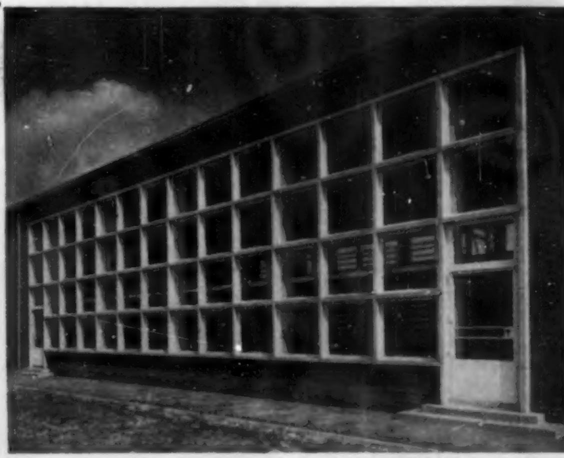

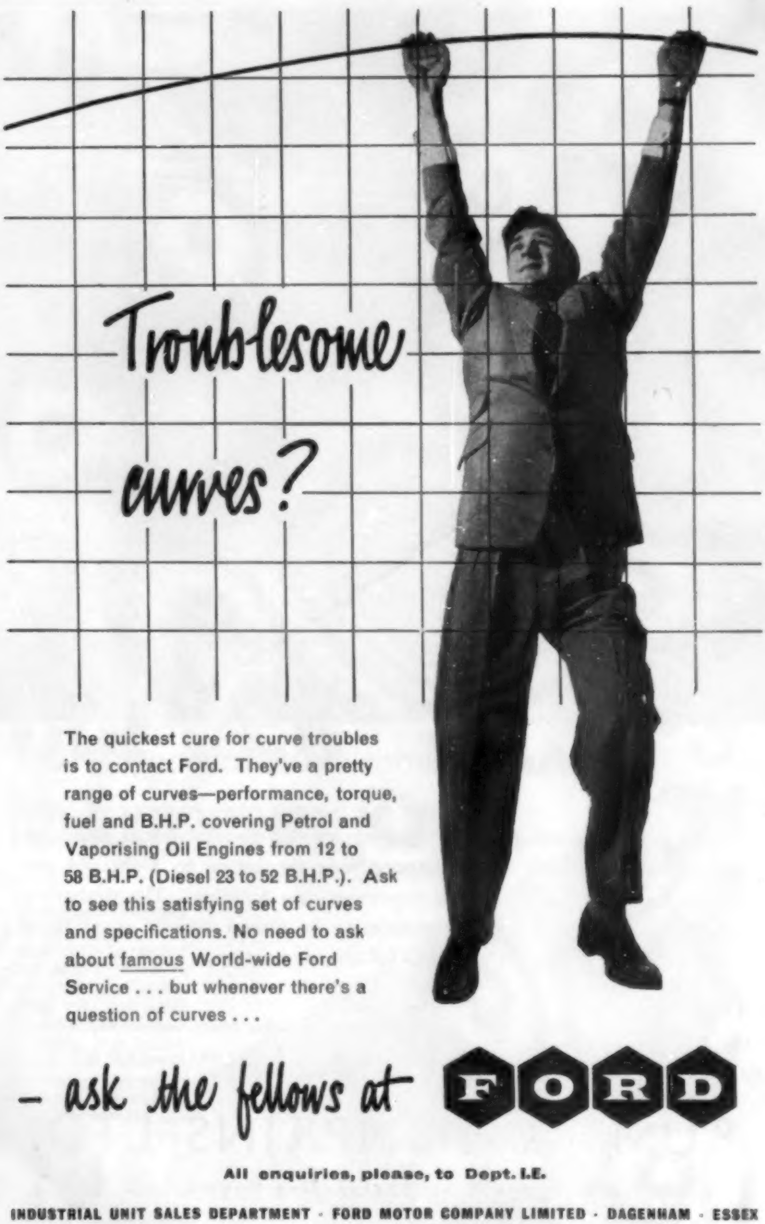
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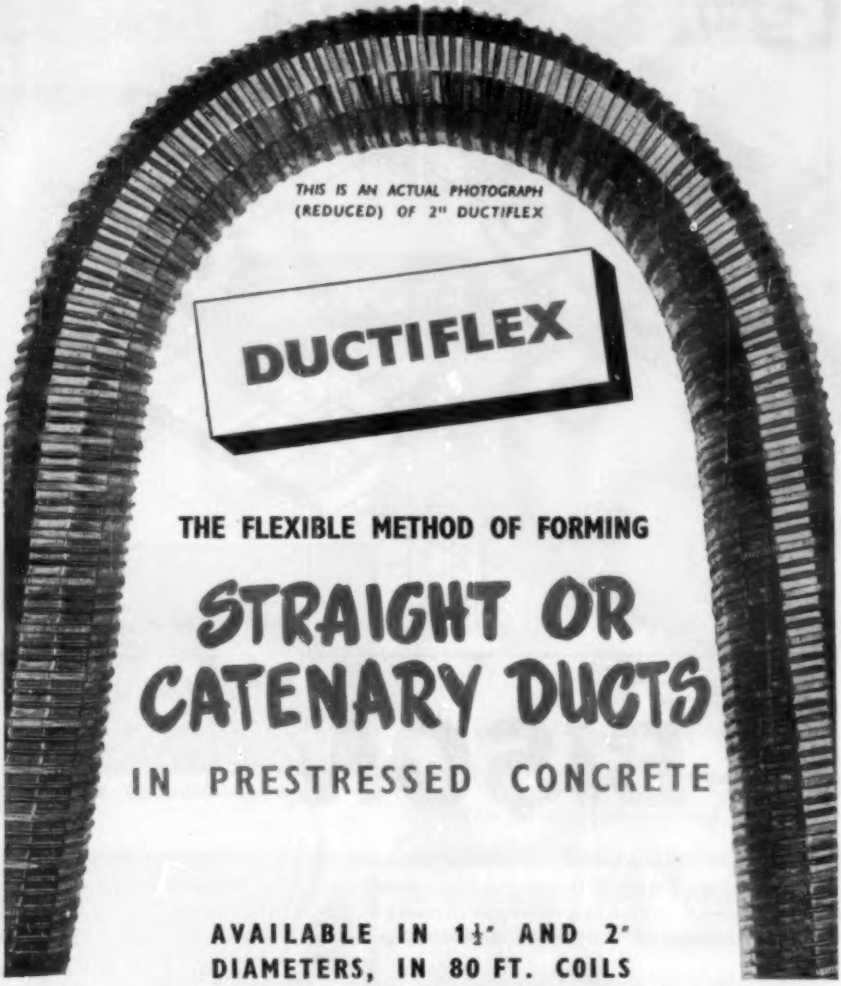

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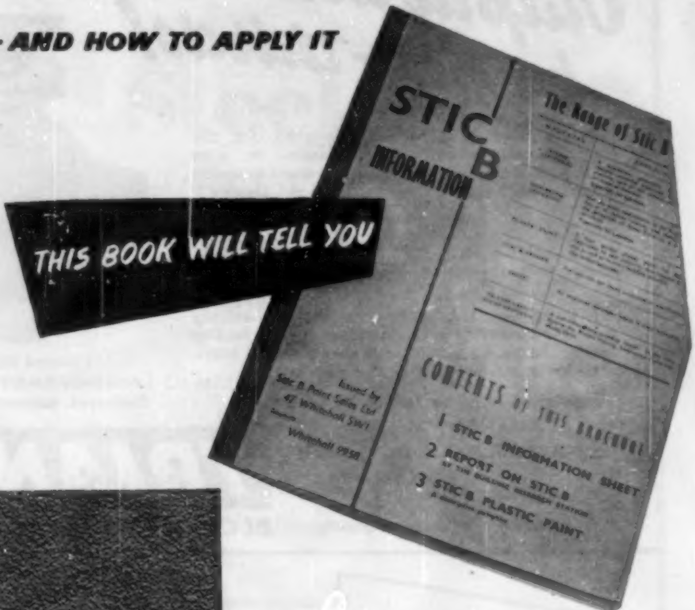

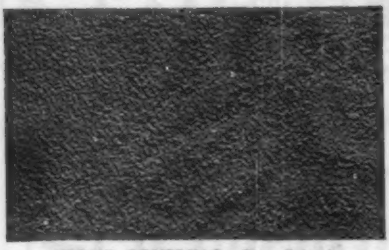
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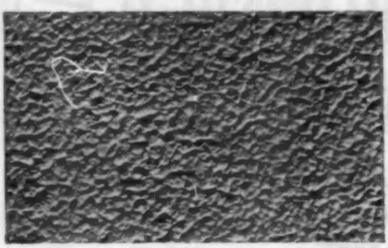
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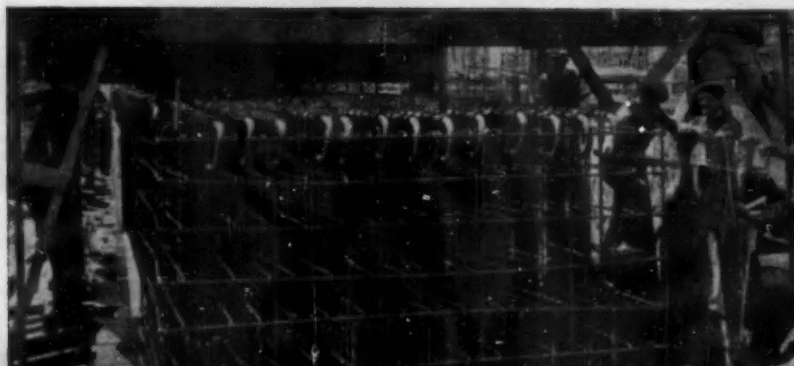

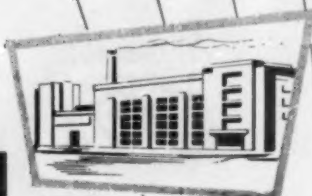

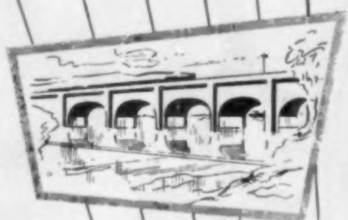

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Volume LI, No. 7. LONDON, JULY, 1956.

EDITORIAL NOTES

By-laws Relating to Fire-resistance.

MUCH time and labour have been wasted in some offices due to lack of knowledge of the regulations relating to the fire-resistance of reinforced concrete contained in Model Building By-laws, Series IV, issued by the Ministry of Health in the year 1953. These by-laws * have been adopted by most local authorities in Great Britain, but it is not uncommon for them to receive for approval designs that do not comply with the requirements for fire-resistance. A common result is that a design that is not acceptable is altered when the requirements are known, and, apart from the time wasted in making the alterations, the revised design is seldom as economical as it would have been had the requirements for fire-resistance been complied with in the first place. For example, designs so altered have incorporated slabs that are unnecessarily thick, or the reinforcement has been so closely spaced that the cost of construction has been increased. It is therefore desirable that the by-laws be studied before a design is made. It may be mentioned that Constructional Bye-laws, Schedule IV, of the London County Council are very similar to the Model Bye-Laws of the Ministry of Health.

The regulations on fire-resistance apply to buildings according to their use, their height, their cubic capacity, their floor area, and in certain classes of buildings the distance from the nearest boundary of adjoining property has to be taken into account. Fire-resistance is measured by the period which an element of a building can withstand heat of a certain intensity and under certain conditions as described in British Standard No. 476, " Fire Tests on Building Materials and Structures ". The requirements vary from half an hour for domestic buildings less than 75 ft. high and up to 125,000 cu. ft. in capacity to four hours for storage buildings exceeding 75 ft. in height, or 7500 sq. ft. in floor area, or 250,000 cu. ft. in capacity. Also, the requirements for the fire-resistance of walls separating buildings vary from one to six hours, and fire-resisting dividing walls must be able to resist fire for from two to four hours. A further by-law states that any part of a structural frame, or any beam or column carrying an external wall, a wall separating buildings, or a fire-division wall, must have the same resistance to fire as the class of building of which it is a structural member.

In the case of reinforced concrete beams and columns, and floors comprising solid or hollow precast beams or hollow blocks, the by-laws include tables giving

* H.M.S.O. Price 2s. 6d.

the minimum dimensions, thicknesses, and cover of concrete, and it is to be noted that the minimum overall dimensions of free-standing columns are 12 in., 10 in., and 8 in. for fire-resistances of two hours, one hour, and half an hour respectively. However, columns with minimum dimensions of 12 in. and 10 in. are permissible for periods of fire-resistance of four hours and two hours respectively provided that 2-in. mesh reinforcement is placed centrally in the cover to the longitudinal reinforcement. This requires some means of holding the mesh in its correct position. One method is to wire concrete half-washers of the required width to the main longitudinal bars and to wire the mesh to the washers. Other methods may be devised, such as steel washers threaded on to the bars and wired to them at intervals, but these may cause difficulty in placing the concrete in the shutters and in keeping the mesh in position, especially if vibrators are used.

In the case of reinforced concrete beams the least covers of concrete required for fire-resistances of four, two, one, and one-half hours are 2½ in., 2 in., 1½ in., and 1 in. respectively. Thus a beam with a fire-resistance of four hours will have an effective width 5 in. less than its total width, instead of the normal 2 in. (1 in. of cover or twice the diameter of the bar on each side); the effective depth will be probably at least 3 in. less than the overall depth, with the possibility of still further reduction due to the lower bars having to be in two layers because of the increased side cover, with a consequently further reduced carrying-capacity.

In the case of solid slabs the normal ½ in. of cover is sufficient except for a fire-resistance of four hours. The minimum floor thicknesses vary from 3 in. to 6 in., and this also applies to precast beams of U, channel, or tee sections, whereas for hollow-block construction the total thicknesses of combustible material (excluding ceiling finishes) vary from 2½ in. to 5 in.; it is, however, possible to obtain the required thickness by a topping which must, of course, be allowed for in the design. In special cases a certificate may be obtained for a special form of design if the components are submitted to the Fire Research Station for testing. For example, an 8-in. square column has been approved for a fire-resistance of one hour with 2-in. mesh reinforcement in the cover. Steel stanchions and beams must also have a cover of concrete varying with the period of fire-resistance required, although brickwork and fire-resisting materials may in certain cases be used in place of concrete. Also concrete columns protected by brickwork may be smaller in cross section than columns not so protected.

It is seen from the foregoing that the designer must be acquainted with the requirements of the by-laws because they affect the thickness of the concrete cover of beams and slabs and the cross-sectional area of columns. In the fourth schedule of the by-laws various constructional materials are listed and their periods of fire-resistance are given. In the regulations of the London County Council a maximum fire-resistance of four hours only is required for walls separating buildings and the minimum requirements for construction and materials are given in the sixth schedule; this compares with six hours in the fourth schedule of the model by-laws of the Ministry of Health. Also, the London County Council regulations give requirements for the fire-resistance of stairs, doors, and glazing that are not specifically dealt with in the by-laws issued by the Ministry of Health. It should be noted that some materials that are generally regarded as fire-resisting may not satisfy the requirements; for example, asbestos-cement sheets may not be accepted as a wall covering for structures near the boundaries of a site.

A Prestressed Dam in Scotland.

An unusual use of prestressed concrete is being made in the Allt-na-Lairige dam at present under construction for the North of Scotland Hydro-electric Board. Allt-na-Lairige is a tributary of the river Fyne in Argyll, and the reservoir formed by the dam will impound 800,000,000 gallons of water. The dam is 1360 ft. long and has a maximum height of about 73 ft. The spillway, draw-off sections, and the low terminal blocks have the usual section of gravity dams but the central section, 966 ft. long, is prestressed. *Fig.* 1 shows the dam during construction and *Fig.* 2 is a typical cross section.

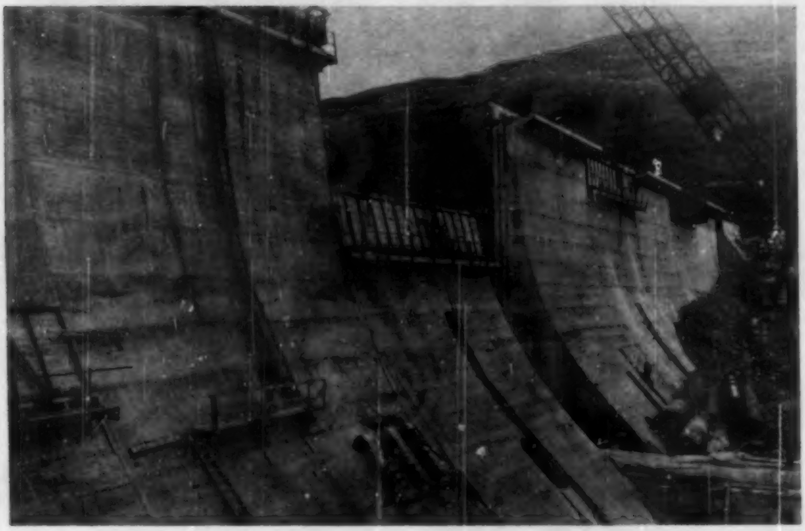


Fig. 1.—Dam during Construction.

The saving compared with the usual type of gravity dam is due to the provision of a vertical stabilising force by prestressed bars at about half the cost of obtaining a similar force by a weight of concrete alone, and also this force may be placed in the most effective positions. The thinner section thus achieved produces further economies by reducing the uplifting force, requiring less excavation, and shortening the period of construction.

In prestressed construction the quantity of concrete is usually small and is highly stressed. In a dam, however, where its own weight produces a stabilising force, the most economical design is related to the volume of concrete and the cost of the high-tensile steel bars and anchorages to produce the desired prestressing force. At Lairige this resulted in a cross section in which about 44,000 tons (24,000 cu. yd.) of concrete and 48,000 tons of vertical prestressing force stabilise the dam. For practical reasons there is a constant prestressing force throughout the full height without intermediate anchorages. This allowed uninterrupted concreting without special scaffolding to support the bars, and all the tensioning was done at the top of the dam after the concrete had been completed.

The criterion for the assumed stresses differs from the usual practice for gravity dams by providing for the full uplifting pressure at the upstream toe and permitting in the lower half of the downstream face a principal tension of 50 lb. per square inch with the reservoir empty.

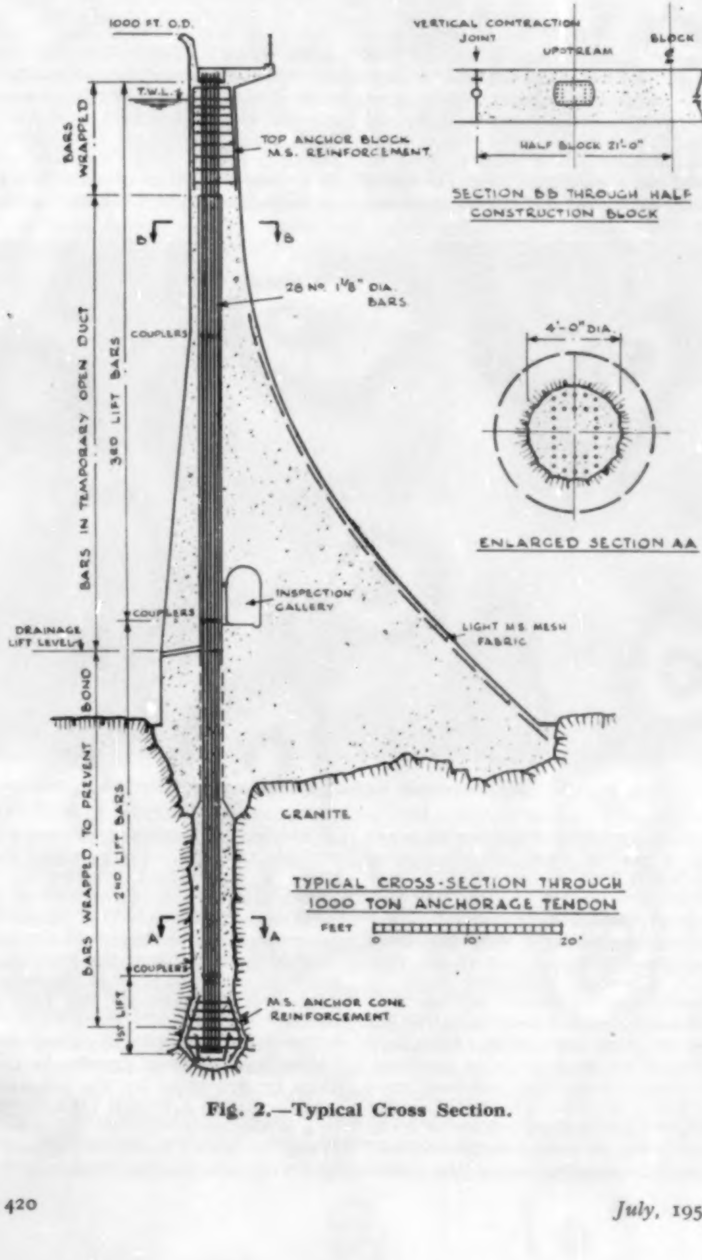


Fig. 2.—Typical Cross Section.

Fig. 3.—Assembly of Prestressing Bars.

Since all the stresses at the base are low this permitted a considerable increase in the effective lever arm of the steel at the base, with consequent saving in steel. A small amount of mesh reinforcement is provided to prevent cracking due to tensile stresses in the downstream face, and the anchorages at the top and bottom are reinforced to resist tension due to the high bearing-pressure (2500 lb. per square inch) of the anchor-plates.

At the beginning of the work tests were made to determine the depth of anchorage required in the rock and to select a suitable non-bonding wrapping for the prestressing bars. Freyssinet flat jacks were cast in the concrete at the bottom of the test-pit and a vertical load of 4400 tons was produced.

Method of Construction.

The dam was constructed in blocks 42 ft. long each with two anchorages in the foundation rock at 21 ft. centres. An anchorage was formed by excavating a pit, 4 ft. diameter and about 27 ft. deep, in the granite below the level of the cut-off trench. The bottom of the pit was conical in shape to form the anchorage. In each anchorage there are twenty-eight $1\frac{1}{8}$-in. diameter high-tensile steel bars which were tensioned to 42 tons per square inch and then released to produce a working stress of 37 tons per square inch, resulting in a force of about 1040 tons per anchorage. The twenty-eight bars are anchored at each end by nuts bearing on a single mild steel plate. The bars were arranged to allow access to the centre for erection and concreting in the pits.

The bars were erected in three lifts (*Fig.* 2). The first two were below the level of the rock and these bars were concreted in the pits and in the foundation of the dam. These bars were covered with tape impregnated with petroleum

Fig. 4.—Prestressing Bars for Third Lift.

grease over which was wrapped bituminous tape for protection during handling and concreting. A group of bars for the first lift is shown in *Fig.* 3. This assembly was lowered by a derrick to the bottom of the pit, and high-quality concrete in the proportions of 1 : 4 was placed in the cone to surround the bars and the anchor-plate. After concreting, the top template was removed and the bars of the second lift were connected. These were then slightly tensioned on a temporary cross-head by tightening the nuts to position and straighten the bars, and the concreting was completed to the level of the drainage channel (*Fig.* 2). Above this level the bars were not covered but were contained in an open shaft measuring 4 ft. by 2 ft. 3 in., which remained open until the tensioning was completed. This shaft starts at the lowest level of the block where it can be drained by gravity.

Concreting was continued to 13 ft. below the top of the dam, the third lift of bars was lowered down the shaft by the derrick, and concrete placed to that level. These bars were suspended from a steel trestle (*Fig.* 4) while being connected to the bars below, and were also slightly tensioned while the next lift of concrete was placed around the bars, which were wrapped to prevent bond. Two more lifts of concrete completed the top anchorage-block and the anchor-plate was then bedded down on 1 : 3 concrete made with granite aggregate. Through the centre of the top anchor-plate there is a duct of 12-in. diameter connecting with the shaft 13 ft. below to enable this to be filled with concrete after tensioning the bars.

Fig. 5.—Tensioning the Bars.

Except at the end anchorages all the stresses in the concrete are low. At the

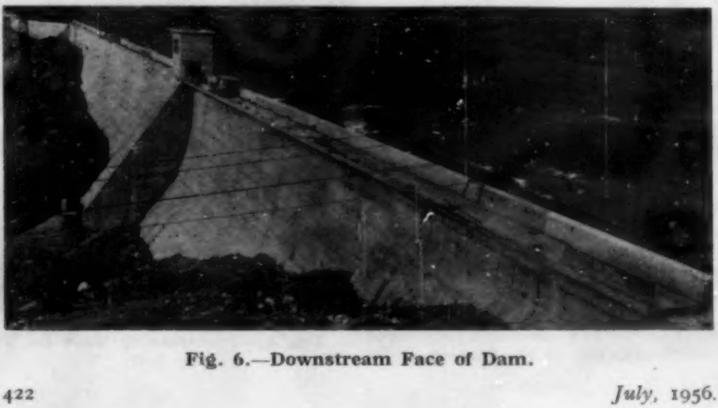


Fig. 6.—Downstream Face of Dam.

bottom the concrete is in the proportions of 1 : 4, and 1 : 5 concrete is used in the top. The concrete in the base is in the proportions of 1 : 8½ in the heart and 1 : 5 in the facings. The concrete was placed in lifts of 5 ft. by derricks. Granite aggregate from 2½ in. down was quarried and crushed on the site, proportioned by weight, and mixed in a 1 cu. yd. mixer. The sand obtained from the crushed rock was mixed with an equal quantity of natural sand, and workability was improved by the addition of an air-entraining agent. The concrete in the anchor-pits was placed by a tremie pipe, and a similar arrangement is now being used to place the concrete in the shafts, in which a very workable mixture in the proportions of 1 : 5½ is used.

The tensioning of the bars at the top anchor-plate is shown in *Fig.* 5, and *Fig.* 6 is a general view of the dam with the main concrete work completed. The remaining work is due to be completed soon, and impounding of water will then begin.

The engineers for the project are Messrs. Babtie, Shaw & Morton, of Glasgow, and the contractors are Messrs. Marples, Ridgway & Partners, Ltd., London. The Lee-McCall system of prestressing was used.

A Dome of 165 ft. Diameter.

A DOME over a public building in Japan is described by Professor Yoshikatsu Tsuboi and Mr. Kinji Akino in Vol. 15 of the Proceedings of the Association for Bridge and Structural Engineering.*

The building (*Fig.* 1) has seating accommodation for 1400 people. The radius of the dome is 164 ft. and the rise 22 ft. The height from the ground to the crown is 46 ft. The roof is supported on steel rollers (*Figs.* 2 and 3) on twenty columns. The dome is 4¾ in. thick at the centre increasing to 2 ft. 3½ in. at the edge. The floor was concreted after the foundation was completed and on this the centering was built. In the centre the supports were spaced 5 ft. 2 in. radially from the apex, and the spacing was reduced as the load increased.

The reinforcement was very closely spaced at the circumference, and was butt-welded and tested in tension before use. Blocks of concrete were fixed to the top and bottom bars to ensure the re-

* Verlag Leeman, Zürich. Price 38 Swiss francs.

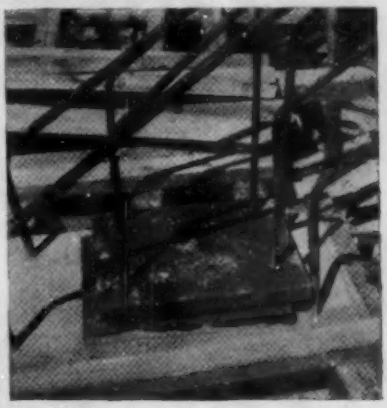


Fig. 2.—Detail of a Roller Bearing.

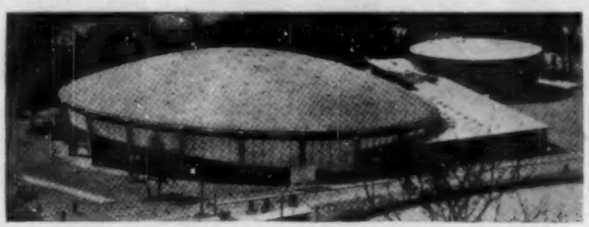


Fig. 1.—Completed Structure.

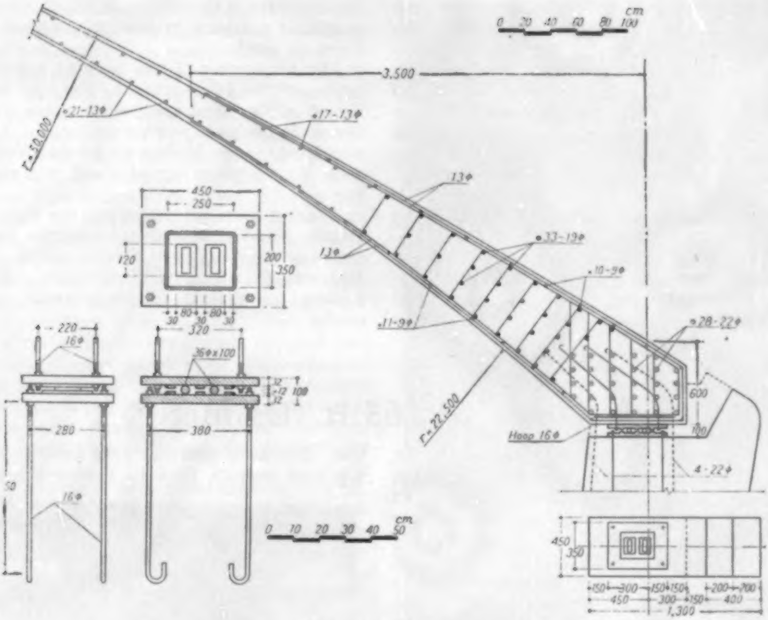


Fig. 3.—Roller Bearing at Circumference.

quired cover. A coarse cement was used in order to reduce cracks due to shrinking. Where the slope was steep, wire mesh was placed on the reinforcement to prevent the concrete sliding. No top shutters were used.

The dome required 733 cu. yd. of concrete. The outer part, containing 490 cu. yd., was cast continuously. Two mixers of 21 cu. ft. capacity were used and concreting was completed in about twenty-three hours. The concrete was compacted by a vibrator in the edge-beam and by hand elsewhere; it was cured with wet mats for four weeks, when the centering was removed.

The central part of the dome is of uniform thickness and was designed by an approximate method. The paper shows how the equations were derived; these include the effects of temperature, unsymmetrical loading, and openings in the slab. The lower part of the dome was designed as a spherical shell of varying thickness, and the method is explained in detail. The results are compared with those obtained by the general approximate method, assuming a uniform thickness.

Stresses in Chimneys due to Temperature.

By NARAIN V. HINGORANI, B.Eng.

In Combination with Vertical Stresses.

AT a section of a chimney the stresses in the concrete and the reinforcement due to the total weight and the wind pressure are first calculated, and the stresses due to the difference in temperature through the thickness of the concrete are then calculated. In the following, equations are derived for combining these stresses, and graphs are given to facilitate the calculations. The following applies to a chimney with reinforcement in the outer face only. The symbols used are :

d, thickness of the wall.
nd, distance of the reinforcement from the inner face.
p, percentage of reinforcement. p', ratio of the area of reinforcement to the area of concrete.
c_w, stress in the concrete due to the weight and the wind.
t_w, stress in the reinforcement due to the weight and the wind.
P, total thrust on the section.
F, total tension in the section.
c_T, stress in the concrete due to fall of temperature across an uncracked section.
c, maximum compressive stress in the concrete due to the weight, wind, and temperature.
t, maximum stress in the reinforcement due to weight, wind, and temperature.
R, mean radius of the chimney.
S, shearing force. c_s, shearing stress in the concrete. t_s, shearing stress in the reinforcement.
a, lever arm coefficient for a horizontal section.
e, eccentricity.

$$K = \frac{c}{c_T}. \quad C = (2n - K). \quad Q = \frac{t_w}{c_T}. \quad U = \frac{c_w}{c_T}.$$

The stresses in the concrete and reinforcement can be calculated for all cases, but the design will be governed by the worst cases, namely (i) compression of the concrete on the leeward side and (ii) tension in the reinforcement on the

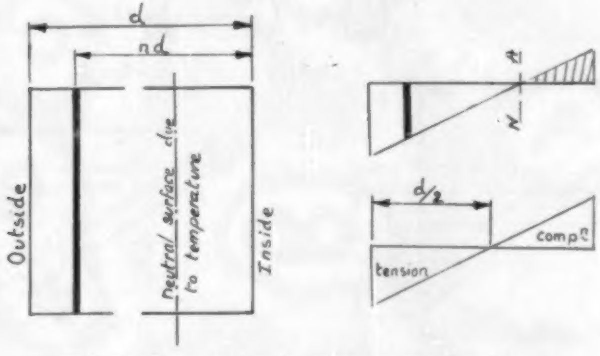


Fig. 1.—Stresses due to Temperature Gradient.

windward side. It is assumed that the ratio of the thickness of the wall to the radius is very small and that all the tensile forces are resisted by the reinforcement.

An elemental vertical section is shown in *Fig.* 1. Due to the hot gases the inner surface becomes hot and there is a temperature gradient (T deg. Fahr.) between the inner and outer faces. As the expansion of the concrete at the inner face is restrained it is compressed and produces tensile forces in the reinforcement and in the concrete in the cooler parts such that the total forces and bending moments remain constant and the section remains plane and horizontal.

TEMPERATURE GRADIENT.—The bending moment in the section is $M = \frac{\alpha EIT}{d}$, where α is the coefficient of thermal expansion and E the modulus of elasticity. Since $c_T = \frac{My}{I}$, $c_T = \alpha ET\frac{y}{d}$. If the neutral axis passes through the centre of the section, $y = \frac{d}{2}$. Therefore $c_T = \frac{\alpha ET}{2}$. This is not the actual stress in the concrete due to the temperature gradient because, due to the presence of reinforcement, the neutral axis will be farther from the steel. It indicates, however, the rate of change of temperature stress in the section.

LEEWARD SIDE.—From *Fig.* 2, the thrust on the section per inch of length, neglecting the effect of temperature, is

$$c_w d[1 + p'(m - 1)] \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The force in the reinforcement when subjected to high temperature is

$$\left(\frac{2nd}{d}c_T - c\right)mp'd \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The reinforcement may be in tension or compression and, since the stress is small, the same modular ratio m is used in both cases. The error in the compressive stress in the concrete will not be very great.

The thrust on the section, including the effects of temperature, equals the thrust on the concrete less the tensile force in the reinforcement, that is,

$$P = \frac{1}{2}\frac{d}{2}\left(\frac{c}{c_T}\right)^2 c_T - \left(\frac{2nd}{d}c_T - c\right)mp'd \quad . \quad . \quad . \quad (3)$$

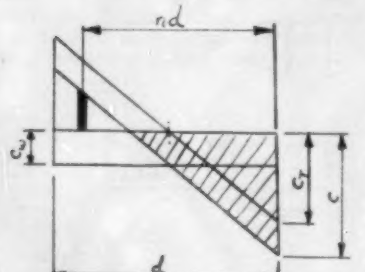


Fig. 2.—Stresses in Leeward Side.

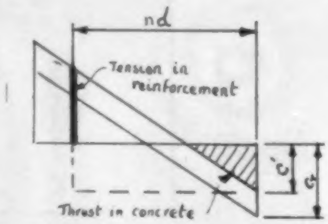


Fig. 3.—Stresses in Windward Side.

For equilibrium, (1) — (3) = 0; that is

$$\frac{c^2}{4c_T} - (2nc_T - c)mp' - c_w(1 + p'(m-1)) = 0 \quad . \quad . \quad (4)$$

or

$$\frac{c^2}{4c_T} + cmp' - [2nc_Tmp' + c_w(1 + p'(m-1))] = 0.$$

Solving this equation,

$$c = 2c_T\left[-mp' + \sqrt{m^2p^2 + 2nmp' + \frac{c_w}{c_T}(1 + p'(m-1))}\right].$$

Assuming that $m = 15$ and $\frac{c_w}{c_T} = U$,

$$c = 2c_T[-15p' + \sqrt{225p'^2 + 30np' + u(1 + 14p')}] = Kc_T.$$

$$t = (2n - K)\,c_Tm = mCc_T = 15Cc_T.$$

Within limits, K is affected only negligibly by the amount of reinforcement. Graphs are given for $p = 0{\cdot}5$, which gives fairly accurate results for up to 1 per cent. of reinforcement. The amount of reinforcement used is generally between 0·3 per cent. and 0·7 per cent.

WINDWARD SIDE.—The initial tensile force in the section is

$$t_wp'd \quad . \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

The tensile force in the section, including the effects of temperature, equals the tension in the reinforcement less the thrust on the concrete. The tensile force in the reinforcement (*Fig.* 3) is

$$\left(\frac{2nd}{d}c_T - c'\right)mp'd \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

The final tensile force is

$$\left(\frac{2nd}{d}c_T - c'\right)mp'd - \frac{1}{2}\frac{d}{2}\left(\frac{c'}{c_T}\right)^2 c_T \quad . \quad . \quad . \quad . \quad (3a)$$

For equilibrium (1*a*) — (3*a*) = 0, that is

$$t_wp'd - \left(\frac{2nd}{d}c_T - c'\right)mp'd + \frac{d}{4}c_T\left(\frac{c'}{c_T}\right)^2 = 0,$$

or

$$\frac{c'^2}{4c_T} + c'mp' + t_wp' - 2nmp'c_T = 0 \quad . \quad . \quad . \quad (4a)$$

Solving (4*a*)

$$c' = 2c_T\left(-mp' + \sqrt{m^2p'^2 - \frac{t_wp'}{c_T} + 2nmp'}\right).$$

If $\frac{t_w}{c_T} = Q$ and $m = 15$,

$$c' = 2c_T[-15p' + \sqrt{225p'^2 + 30np' - Qp'}] = K'c_T.$$

$c' = 0$ when $30np' = Qp'$ or $Q = 30n$, or $\dfrac{t_w}{c_T} = 30n$, that is when $t_w = 30nc_T$.

$$t = (2n - K')c_T m. \quad \text{If } c' = 0 \text{ then } K' = 0, t = 2mnc_T = 30\ nc_T.$$

Hence if, due to the dead load and the wind only, $t_w = 30\ nc_T$, there will be no change in the stress in the reinforcement regardless of the difference of temperature across the wall.

If the reinforcement is neglected in equation (4), then

$$\frac{d}{4}\frac{c^2}{c_T} = c_w d, \text{ or } c^2 = 4c_w c_T.$$

c can be a maximum when $c_w = c_T$. Therefore $c = 2c_w$ or $2c_T$ or $c_w + c_T$. When the reinforcement is considered the maximum stress in the concrete can be little more than $c_w + c_T$, depending upon the value of n. Hence if c_w and c_T are nearly equal it is possible to estimate the stress in the concrete with reasonable accuracy. The use of these formulæ and graphs makes it unnecessary to calculate the position of the neutral axis due to temperature stresses only.

Use of *Graphs* I and II.

At a certain level a wall is $7\frac{1}{2}$ in. thick and the reinforcement comprises $\frac{1}{2}$-in. bars at 5-in. centres with a cover of $1\frac{1}{2}$ in. to the outer face. At this level c_w on the leeward side is 350 lb. per square inch and t_w on the windward side is 3500 lb. per square inch. Also, when hot gases pass through the chimney the maximum difference in temperature in the concrete is 145 deg. Fahr. The coefficient of expansion of concrete is 6×10^{-6} per deg. Fahr., and the modulus of elasticity is 2×10^6 lb. per square inch. d is $7\frac{1}{2}$ in. Therefore

$$nd = 7\tfrac{1}{2} - (1\tfrac{1}{2} + \tfrac{1}{4}) = 5\tfrac{3}{4} \text{ in., and } n = \frac{5{\cdot}75}{7{\cdot}5} = 0{\cdot}766.$$

The percentage of reinforcement is $\dfrac{0{\cdot}471}{90} \times 100 = 0{\cdot}523$.

$c_T = T\alpha E\dfrac{y}{d} = 135 \times 12 \times \frac{1}{2} = 810$ lb. per square inch. $\left(\dfrac{y}{d} = \frac{1}{2}\right.$ when the section is uncracked.)

$$U = \frac{c_w}{c_T} = \frac{350}{810} = 0{\cdot}432. \quad Q = \frac{t_w}{c_T} = \frac{3500}{810} = 4{\cdot}32.$$

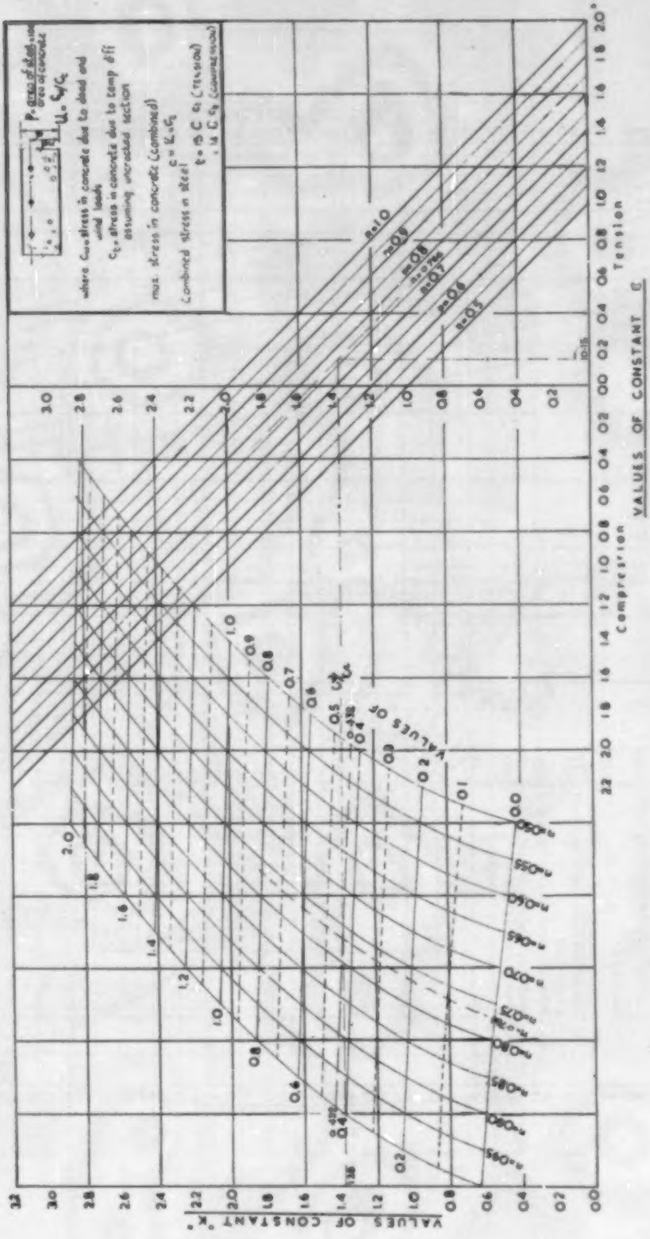
Graph I—(Leeward Side).—For $u = 0{\cdot}432$ and $n = 0{\cdot}766$, $K = 1{\cdot}38$ and $C = 0{\cdot}15$ (positive). $c = 1{\cdot}38 \times 810 = 1118$ lb. per square inch.

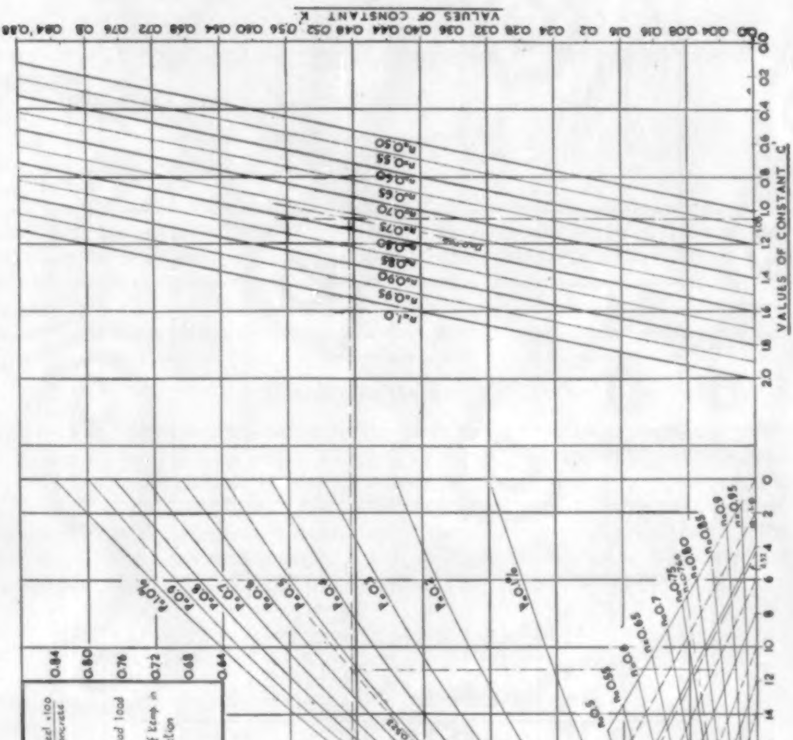
$$t = 15 \times 0{\cdot}15 \times 810 = 1820 \text{ lb. per square inch.}$$

Graph II—(Windward Side).—$Q = 4{\cdot}32$, $p = 0{\cdot}523$, $K' = 0{\cdot}486$, and $C = (2n - K') = 1{\cdot}05$. Therefore $c' = 0{\cdot}486 \times 810 = 394$ lb. per square inch and $t = 1{\cdot}05 \times 15 \times 810 = 12{,}750$ lb. per square inch.

If the reinforcement is in compression on the windward side, t can be obtained from the formulæ for the leeward side and from *Graph* No. I.

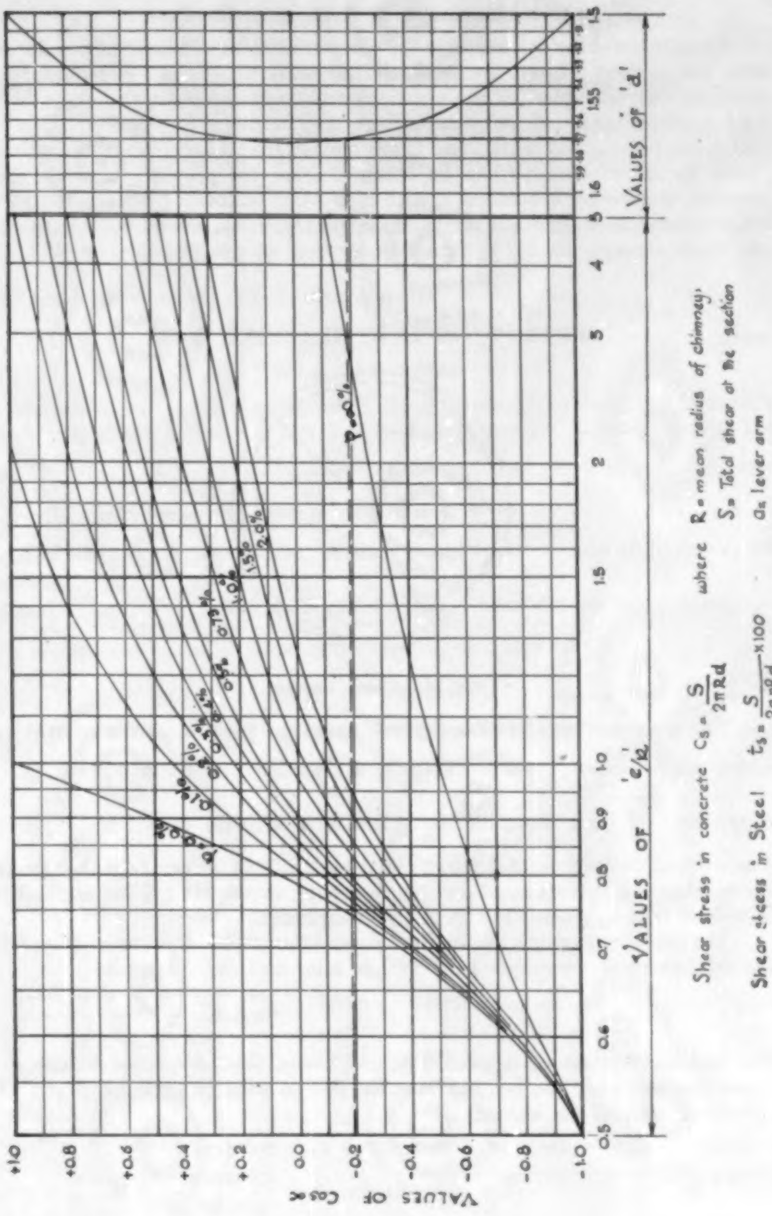
GRAPH I.—STRESSES IN LEEWARD SIDE.





GRAPH II.—STRESSES IN WINDWARD SIDE.

GRAPH III.—SHEARING STRESSES.



In Combination with Horizontal Stresses.

Usually the shearing stresses in the concrete and the circumferential reinforcement due to wind pressure are small, so that when calculating c in the horizontal direction and the stress in the circumferential reinforcement the effect of the wind is often neglected and temperature only is taken into account. In this case it can be assumed that c_w and t_w are zero, so that u and Q are zero and c and t (the maximum stresses) can be obtained from the graphs. In some cases, however, it is better to combine t_s with c_T in order to obtain t. Since, due to the temperature, the concrete is cracked, the reinforcement should be able to resist the whole shearing force; c_s should be as small as possible.

$c_s = \frac{S}{2\pi Rd}$. $t_s = \frac{S}{aR \times 2p'd}$ where p' refers to the circumferential reinforcement.

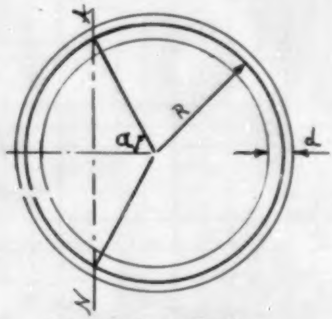


Fig. 4.—Cross Section.

The lever-arm coefficient a depends upon the position of the neutral axis, which depends upon $\frac{e}{R}$ where e is the eccentricity due to the wind (*Fig.* 4). In *Graph* No. III $\cos\alpha$ connects the values of a with the values of $\frac{e}{R}$. In this case α is half the angle subtended by the neutral axis at the centre of the chimney. By substituting this value of a in the foregoing expression t_s is found, and this combined with c_T gives t for the worst conditions.

The following example illustrates the use of the graph. $R = 10$ ft.; $d = 6$ in.; the reinforcement comprises ½-in. bars at 6-in. centres. Therefore

$$p' = \frac{0{\cdot}393}{6 \times 12} = \frac{0{\cdot}393}{72} = 0{\cdot}00546.$$

The load on the chimney is 500,000 lb. and a wind load of 100,000 lb. acts 40 ft. above the section. The bending moment due to wind is 4,000,000 lb.-ft. The equivalent area of the section

$$2\pi Rd(1 + 14p) = 2\pi \times 10 \times 12 \times 6(1 + 14 \times 0{\cdot}00546)$$

$$= 4860 \text{ sq. in.} \quad e = \frac{4{,}000{,}000}{500{,}000} = 8 \text{ ft.} \quad \frac{e}{R} = 0{\cdot}8.$$

$$c_s = \frac{100{,}000}{2 \times \pi \times 10 \times 12 \times 6} = \frac{100{,}000}{4{,}520} = 22{\cdot}1 \text{ lb. per square inch.}$$

$$t_s = \frac{S}{2ap'Rd} = \frac{100{,}000}{2 \times 1{\cdot}57 \times 0{\cdot}00546 \times 120 \times 6} = 8100 \text{ lb. per square inch.}$$

(The value of a is found from *Graph* No. III.) This stress combined with that due to the temperature obtained from *Graph* No. II is the maximum stress in the circumferential reinforcement.

BIBLIOGRAPHY.—" Reinforced Concrete Chimneys ", by Taylor and Turner. " Reinforced Concrete Masonry Structures ", by Hool and Kinne.

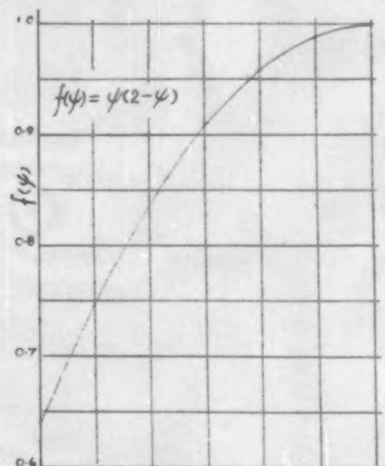
The Design of Tee-beams.

By J. S. SAVONA.

REFERRING to *Fig.* 1, the symbols used are: b_f, breadth of flange; b_w, breadth of web; d_s, depth of slab; d, depth of beam; n, depth to neutral axis; ψ, the ratio $\frac{d_s}{n}$; $b_o = b_f - b_w$; $b_i = f(\psi)b_o$ and $f(\psi) = \psi(2 - \psi)$; $b_e = b_i + b_w$; A_t, area of tensile reinforcement; c, compressive stress.

The two lateral parts of the flange each of breadth $\frac{b_o}{2}$ and thickness d_s are reduced to equivalent breadth $\frac{b_i}{2}$ and depth n. Thus the tee-beam is reduced to a rectangular beam of breadth b_e equal to b_i added to b_w.

GRAPH I.



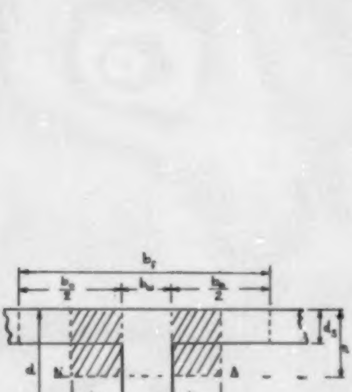


Fig. 1.

The total compression in the parts $\frac{b_o}{2}$ of the flange is

$$\tfrac{1}{2}\left[c + c\frac{(n - d_s)}{n}\right]b_o d_s = \tfrac{1}{2}c\frac{(2n - d_s)}{n}b_o d_s.$$

If b_i is the overall breadth of parts equivalent to $\frac{b_o}{2}$ and of depth n, the total compression is $\frac{1}{2}cb_i n$ (*Fig.* 1), but $\frac{1}{2}cb_i n = \frac{1}{2}c\frac{(2n - d_s)}{n}b_o d_s$. If $\psi = \frac{d_s}{n}$, then $b_i = \psi(2 - \psi)b_o = f(\psi)b_o$ and the breadth of the equivalent rectangular beam is $b_e = b_i + b_w$. The relation between ψ and $f(\psi)$ is shown in *Graph* I.

EXAMPLE 1.—A tee-beam is subjected to a bending moment of 1,600,000 in.-lb.; b_f is 68 in.; b_w is 10 in.; d_s is 3·2 in. If the compressive stress in the concrete is 575 lb. per square inch and the tensile stress in the steel 20,000 lb. per square inch, and the modular ratio is 15, determine the depth of the beam and the area of reinforcement A_t.

Assume that $\psi = 0{\cdot}475$. Then $b_o = b_f - b_w = 68 - 10 = 58$ in. Since $f(\psi) = 0{\cdot}725$, $b_i = f(\psi)b_o = 0{\cdot}725 \times 58 = 42$ in. Therefore $b_e = 42 + 10 = 52$ in. It will be found that d is 19·6 in. and n is 5·9 in., from which ψ is 0·53.

Assuming that $\psi = 0{\cdot}65$, then b_e is 60·8 in., while the corresponding value of d is 18·9 in. and of n 5·73 in. This results in ψ equal to 0·575.

Assuming that ψ is 0·55, b_e is 56·3 in., d is 19·05 in., and n is 5·7 in. This results in an actual value of ψ of 0·557, which is very near the chosen value. The area of the reinforcement may now be calculated, and is 4·62 sq. in. It will be seen that the value of ψ fluctuates between the actual and assumed values until an assumption affords the required solution; two or three trials will suffice.

EXAMPLE 2.—A tee-beam has a breadth of flange of 72 in., d_s is 4 in., d is 24 in., and b_w is 15 in. If the beam has to resist a bending moment of 2,600,000 in.-lb. and the reinforcement consists of ten $\frac{7}{8}$-in. bars ($A_t = 6{\cdot}03$ sq. in.), determine the stresses in the concrete and steel. m is 15.

Assuming that $\psi = 0{\cdot}46$, then $f(\psi) = 0{\cdot}673$. $b_o = 72 - 15 = 57$ in. $b_i = 0{\cdot}673 \times 57 = 38{\cdot}3$ in. $b_e = 38{\cdot}3 + 15 = 53{\cdot}3$ in.

$$p = \frac{6{\cdot}03}{53{\cdot}3 \times 24} = 0{\cdot}00471. \quad mp = 0{\cdot}0708.$$

Hence $\alpha = 7{\cdot}13$*; $n_1 = 0{\cdot}313$, $n = 7{\cdot}52$ in., and $\psi = 0{\cdot}532$.

Assuming that $\psi = 0{\cdot}475$ then $f(\psi) = 0{\cdot}725$, $b_i = 41{\cdot}4$, $b_e = 56{\cdot}4$ in., $p = \dfrac{6{\cdot}03}{56{\cdot}4 \times 24} = 0{\cdot}00446$, and $mp = 0{\cdot}0668$.

Hence $\alpha = 6{\cdot}47$, $n_1 = 0{\cdot}35$, $n = 8{\cdot}4$ in., and $\psi = 0{\cdot}476$.

The last assumption is correct and the stresses are $c = 532$ lb. per square inch and $t = 14{,}880$ lb. per square inch.

*See this journal for May, 1954.

Reservoir with Walls of Novel Shape.

A RESERVOIR with a capacity of 25,000,000 gallons, divided into two unequal parts, is being built at Bushey Heath, Herts, for the Colne Valley Water Company. The western part is 523 ft. long by 238 ft. wide and has a capacity of 14,500,000 gallons. The eastern part is 313 ft. by 285 ft. and has a capacity of 10,500,000 gallons. The average depth of water is 21 ft. in both parts. There is a valve house at one end of the wall between the compartments. The inlet pipes which cross the floor of the reservoir are of reinforced concrete with flexible rubber joints and are anchored to precast concrete cradles. The pipes are 3 ft. diameter in the western part and 2 ft. 6 in. diameter in the eastern part.

The floor is 12 in. thick and reinforced in the top and bottom in both directions. The top reinforcement comprises twisted square bars in order to prevent surface cracks. The floor was laid on a blinding 3 in. thick covered with a single layer of bitumen emulsion to prevent adhesion. The floor was cast in alternate bays each 30 ft. by 15 ft., and the construction joints are mid-way between the centres of the columns. Expansion joints are at intervals not exceeding 100 ft.

The reinforced concrete perimeter walls are of an unusual shape (*Figs.* 1 and 2). A "wall unit" comprises a base-slab 14 ft. wide incorporating a "floor beam" 4 ft. 6 in. deep, a lower section of wall 6 ft. high sloping outwards at 60 deg. and bearing against the excavated face, and an upper section of wall 20 ft. high sloping inwards at 60 deg. and incorporating a beam about mid-height. The floor and wall-beams are connected by sloping reinforced concrete members at 15-ft. centres. Expansion joints are at intervals not exceeding 90 ft. and there is a continuous expansion joint between the base of the wall and the floor-slab.

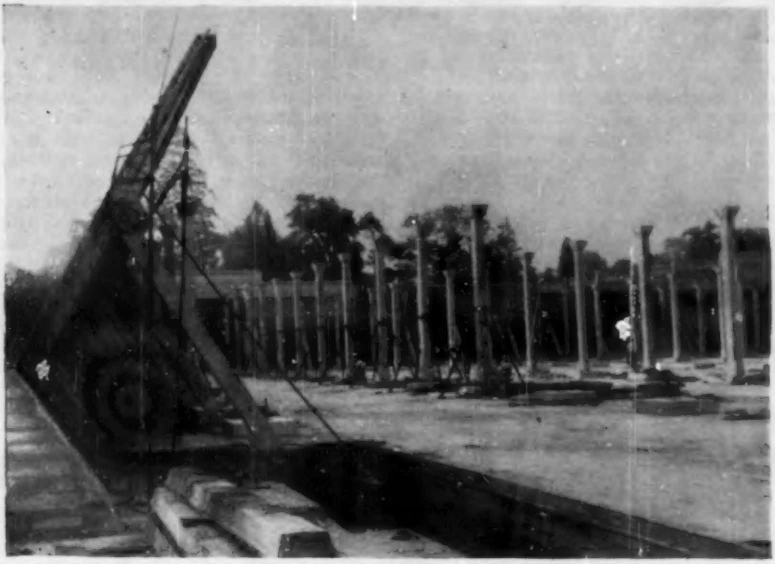


Fig. 1.—Perimeter Wall during Construction.

The wall is designed to resist the full head of water without embankments, the outward hydrostatic thrust being resisted by the lower sloping part of the wall and

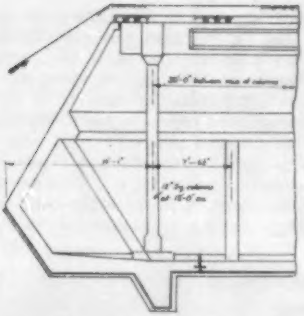


Fig. 2.—Section Through Perimeter Wall.

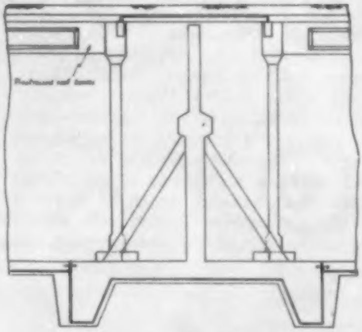


Fig. 3.—Section Through Dividing Wall.

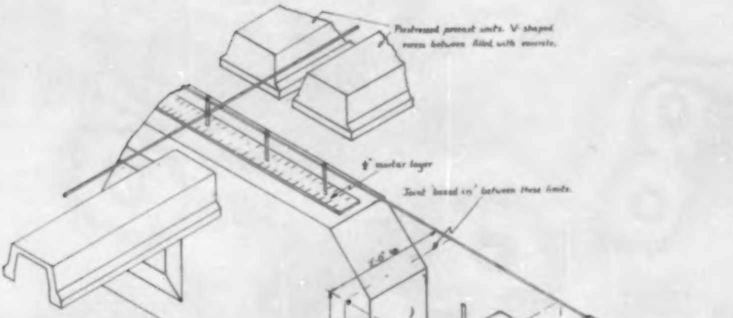


Fig. 4.—Arrangement of Precast Beams and Roof Units.

the floor-beam. The weight of earth on the top inwardly-sloping section of wall is slightly greater than the outward force due to the water pressure, and therefore the inner face of the wall will always be in compression. Other reasons for adopting this shape of wall are that on a restricted site additional storage capacity is obtained and less material is required for embankments. The extra storage capacity in the "bulge" of the wall is about 1,500,000 gallons, and the saving in material required for embankments amounted to about 10,000 cu. yd. The surplus excavated material was used for the construction of an earthen dam elsewhere in the Company's area.

The total length of the perimeter wall is more than 2000 ft., and therefore it was economical to provide special shuttering. The wall was constructed in sections 30 ft. long, each being built in four lifts. The first lift comprised the base-slab and the floor-beam, and included the building in of the raking struts which were precast with reinforcement protruding at each end. The second lift was the lower outwardly sloping part, followed by the inwardly-sloping part up to and including the wall-beam. The final stage was the top part above the wall-beam.

The dividing wall (*Fig.* 3) is vertical and at mid-height is strengthened by a beam tied to the floor-beams on either side by reinforced concrete members at 15-ft. centres. As in the case of the perimeter wall, the raking struts were precast and built into the base-slab.

The roof is formed almost entirely of precast members. The column bases were cast in situ on the floor slab, and on these precast columns 12 in. square were erected. The main roof beams are prestressed precast members 30 ft. long and are supported on precast column caps. The heads of the columns were cast after the beams had been placed in position.

The roof comprises prestressed precast trough units 15 ft. long supported by the beams (*Fig.* 4). The gaps between the sloping sides of the roof units were filled with concrete which incorporated reinforcement over the top of the main beams to produce continuity.

All the prestressed roof members were designed so that their bottom surfaces are in compression under the dead load, which permits the lower part of the main beams to be submerged when the reservoir is full and reduces the total height required.

The roof is waterproofed with three layers of bituminous emulsion reinforced with two layers of glass fibre. Soil, 12 in. deep, is spread over the top of the roof and sown with grass. The floor has a slight fall towards the valve-house, and as the columns and perimeter wall are of equal height there is a slight fall on the roof to facilitate drainage.

The consulting engineers for the reservoir are Messrs. John Taylor & Sons, assisted by Messrs. Leslie Turner & Partners who gave advice on structural design. The contractors were Messrs. James Miller & Partners, Ltd. The contract sum was £315,610 which included all the work apart from bulk excavation and the concrete blinding layer, which had been previously laid by Messrs. W. & C. French, Ltd. All the precast reinforced and prestressed concrete units except the roof units were made by Shockcrete, Ltd.

Pulverised-fuel Ash in a Scottish Dam.

In the construction of Lednoch dam, near Comrie, Scotland, now in course of construction, about 100,000 cu. yd. of concrete will be used. The specified compressive strength is 2800 lb. per square inch at 28 days. The proportions are generally 1 part of cement to 10 parts of aggregate. Pulverised-fuel ash from the North of Scotland Hydro-electric Board's power station at Barrhead is being used in replacement of 20 per cent. of the cement content. Similar concrete is being used in the construction of a dam at Glen Lyon. We are informed that the specified compressive strength has been obtained without difficulty.

Symposium on the Strength of Concrete Structures.

A SYMPOSIUM on the strength of concrete structures, arranged by the Cement and Concrete Association and the Joint Committee on Structural Concrete, was held in London in May. The following papers were presented.

"Some Results of the Theory of Probability in the Estimation of Design Loads", by M. R. Horne.

"The Determination of the Design Factor for Reinforced Concrete Structures", by Arne I. Johnson.

"Current Trends in the Specification of Structural Safety", by Professor Sir Alfred G. Pugsley, O.B.E.

"The Strength of Singly-reinforced Beams in Bending", by A. H. Mattock.

"The Strength of Concrete Members in Combined Bending and Torsion", by S. Armstrong.

"Strength of Prestressed Concrete Members", by Professor C. P. Siess.

"Moment Redistribution in Continuous Beams Reinforced with Plain and Deformed Bars", by K. Hajnal-Kònyi and H. E. Lewis.

"The Failure of Concrete under Compound Stress", by A. J. Harris.

"Ultimate Load Design of Reinforced and Prestressed Concrete Frames", by Professor A. L. L. Baker.

"The Strength of Statically-indeterminate Prestressed Concrete Structures", by Y. Guyon.

"The Strength of Prestressed Concrete Continuous Beams and Simple Plane Frames", by P. B. Morice and H. E. Lewis.

"The Strength of Concrete Walls under Axial and Eccentric Loads", by A. E. Seddon.

"The Strength of Concrete Members under Dynamic Loading", by S. C. C. Bate.

"The Strength of Simply-supported Slab Bridges subjected to Concentrated Loads", by P. B. Morice and G. C. Reynolds.

"Ultimate Strength of Reinforced Concrete in American Design Practice", by Eivind Hognestad.

"The Design of Reinforced Concrete Members", by A. Aas-Jakobsen.

"Load-factor Design in Building Regulations: Future British Practice", by D. D. Matthews.

The Reinforced Concrete Association.

THE annual general meeting of the Reinforced Concrete Association was held in London on June 6. The officers for the ensuing year were elected as follows:

President: Mr. F. S. Snow, O.B.E. (F. S. Snow & Partners).

Vice-President: Mr. E. J. Cook (Richard Costain, Ltd.).

Members: Messrs. E. Bellringer (Stothert & Pitt, Ltd.), E. L. V. Dakin, M.C. (Wilson Lovatt & Sons, Ltd.), P. H. T. Gooding (Cement & Concrete Association), D. P. Harris (Taylor Woodrow Construction, Ltd.), H. S. Killick, M.C. (Twisteel Reinforcement, Ltd.), W. K. Laing (John Laing & Son, Ltd.), A. Raymond Mais, O.B.E. (Trollope & Colls, Ltd.), L. J. Murdock (George Wimpey & Co., Ltd.), A. R. Neelands, M.C. (Cementation Co., Ltd.), H. F. Rosevear (Sir Robert McAlpine & Sons, Ltd.), E. S. Shellard (Associated Portland Cement Manufacturers, Ltd.), C. W. J. Spicer (Considère Constructions, Ltd.), G. W. Stokes (Matthews & Mumby, Ltd.), F. W. Sully [Holloway Bros. (London), Ltd.].

Associates: Messrs. M. G. C. Ayre, C. V. Blumfield, F. C. Eales, A. W. Hill, E. Shepley, J. Singleton-Green, and D. A. Stewart.

Branch Delegates: Messrs. E. J. Martin (Holst & Co., Ltd.), North Western Branch; A. P. Mason (British Reinforced Concrete Engineering Co., Ltd.), Midland Counties Branch; W. A. Fairhurst (F. A. Macdonald & Partners), Scottish branch.

The retiring President (Mr. D. H. New, of Holland & Hannen and Cubitts, Ltd.) presided at the annual luncheon held on the same day, and presented the Association's medal for the past year to Mr. J. A. Derrington for his paper on "Precasting Concrete Structures".

General Formulæ for Maximum Bending Moments.

IMAGINARY DIVERSITY OF LOAD.

By L. S. MÜLLER.

An assumption of an imaginary diversity of load is a useful method of deriving general formulæ for the maximum bending moments of certain continuous structures. If one set of loads is required to cause maximum bending moments at the supports or corners, the application of another, necessarily different, set of loads is required to obtain the maximum moments within the span. By the use of this principle general formulæ may easily be derived in a form in which equating some of the loads (initially assumed as different) to each other, some of them to zero or to the total load, and others to the dead load, will provide formulæ for the different maxima bending moments. This makes it unnecessary to deal separately with dead and live loads.

The principle is explained by two examples chosen to show the application of the method to common problems.

Example 1.—Three Rectangular Tanks or Silos (*Fig.* 1).—In this case the slope-deflection method is used and it is assumed that the pressures are w_1, w_2, and w_3. In order to define the rotations consistently assume that w_1 is greater than w_2 and w_2 is less than w_3. For simplicity, assume that the stiffness of the members is constant, and therefore, the stiffnesses and Young's modulus E may be omitted from the equations.

For point A: $4 \times 2\theta_A + 2\theta_B + 2\theta_H + \dfrac{w_1(l_2^2 - l_1^2)}{12} = 0$

" " B: $4 \times 3\theta_B + 2\theta_A + 2\theta_C + 2\theta_G - \dfrac{(w_1 - w_2)(l_2^2 - l_1^2)}{12} = 0$

" " C: $4 \times 3\theta_C + 2\theta_B + 2\theta_D + 2\theta_F + \dfrac{(w_3 - w_2)(l_2^2 - l_1^2)}{12} = 0$

" " D: $4 \times 2\theta_D + 2\theta_C + 2\theta_E + \dfrac{w_3(l_1^2 - l_2^2)}{12} = 0$

Since $\theta_H = -\theta_A$, $\theta_G = -\theta_B$, $\theta_F = -\theta_C$, and $\theta_E = -\theta_D$, the solutions of the four simultaneous equations are

$$\theta_A = -\frac{81w_1 - 17w_2 + 4w_3}{374} \cdot \frac{(l_2^2 - l_1^2)}{12}. \qquad \theta_B = \frac{56w_1 - 51w_2 + 12w_3}{374} \cdot \frac{(l_2^2 - l_1^2)}{12}.$$

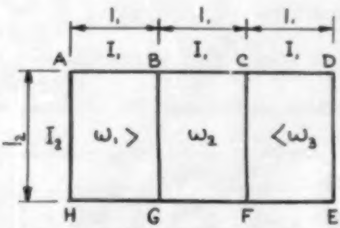


Fig. 1.—Three Rectangular Tanks.

$$\theta_C = -\frac{12w_1 - 51w_2 + 56w_3}{374}\cdot\frac{(l_2^2 - l_1^2)}{12}. \quad \theta_D = \frac{4w_1 - 17w_2 + 81w_3}{374}\cdot\frac{(l_2^2 - l_1^2)}{12}.$$

From these

$$M_{AB} = 2\,(2\theta_A + \theta_B) - \frac{w_1 l_1^2}{12} = \frac{l_2^2 - l_1^2}{187 \times 12}(-106w_1 - 17w_2 + 4w_3) - \frac{w_1 l_1^2}{12}.$$

By substituting

$$\frac{l_2}{l_1} = m, \quad M_{AB} = \frac{1}{187}\cdot\frac{l_1}{12}[-(81 + 106m^2)w_1 + 17(1 - m^2)w_2 - 4(1 - m^2)w_3]$$

Similarly $$M_{BA} = \frac{1}{187}\cdot\frac{l_1^2}{12}[(156 + 31m^2)w_1 + 85(1 - m^2)w_2 - 20(1 - m^2)w_3]$$

$$M_{BC} = \frac{1}{187}\cdot\frac{l_1^2}{12}[-100(1 - m^2)w_1 - (136 + 51m^2)w_2 + 32(1 - m^2)w_3]$$

$$M_{BG} = \frac{1}{187}\cdot\frac{l_1^2}{12}[-(56 + 131m^2)w_1 + (51 + 136m^2)w_2 - 12(1 - m^2)w_3]$$

For the maximum value of M_{AB} and also for the maximum moment on the span AH, $w_1 = w_3 = w$, and $w_2 = 0$. $M_{AB}(\text{max.}) = M_{AH}(\text{max.}) = -M_{HA}(\text{max.})$

$$= -\frac{1}{187}\cdot\frac{wl_1^2}{12}(85 + 102m^2) = -wl_2^2\left(\frac{1}{22} + \frac{1}{26{\cdot}4m^2}\right)$$

Therefore, for member AH

$$+M_{(max.)} = \left[\frac{1}{8} - \left(\frac{1}{22} + \frac{1}{26{\cdot}4m^2}\right)\right]wl_2^2 = \frac{2{\cdot}1m^2 - 1}{26{\cdot}4m^2}.wl_2^2.$$

For maximum moment within the span AB the conditions are as before, so that

$$M_{AB} = -\left(\frac{m^2}{22} + \frac{1}{26{\cdot}4}\right)wl_1^2$$

and $$M_{BA} = \frac{1}{187}\cdot\frac{wl_1^2}{12}(136 + 51m^2) = \left(\frac{1}{16{\cdot}5} + \frac{m^2}{44}\right)wl_1^2,$$

from which $$+M_{(max.)} = \left(\frac{1}{13{\cdot}2} - \frac{m^2}{28{\cdot}8} + \frac{m^4}{3880}\right)wl_1^2$$

For $M_{BA}(\text{max.})$, $w_1 = w_2 = w$ and $w_3 = 0$.

Therefore $$M_{BA}(\text{max.}) = \frac{1}{187}\cdot\frac{wl_1^2}{12}(241 - 54m^2) = \left(\frac{1}{9{\cdot}3} - \frac{m^2}{41{\cdot}5}\right)wl_1^2$$

and with the same conditions,

$$M_{BC}(\text{max.}) = -\frac{1}{187}\cdot\frac{wl_1^2}{12}(236 - 49m^2) = -\left(\frac{1}{9{\cdot}5} - \frac{m^2}{45{\cdot}8}\right)wl_1^2$$

For maximum moment on the span BC, $w_1 = w_3 = 0$, and $w_2 = w$.

Therefore $$M_{BC} = M_{CB} = \mp\frac{1}{187}\cdot\frac{wl_1^2}{12}(136 + 51m^2) = \mp\left(\frac{1}{16{\cdot}5} + \frac{m^2}{44}\right)wl_1^2,$$

so that $$+M(\text{max.}) = \left(-\frac{1}{8} - \frac{1}{16{\cdot}5} - \frac{m^2}{44}\right)wl_1^2 = \left(\frac{1}{15{\cdot}5} - \frac{m^2}{44}\right)wl_1^2$$

For maximum moment in the span BG:

(a) If $m \leq 1$, $w_1 = w_3 = 0$ and $w_2 = w$.

Therefore $$-M_{GB} = M_{BG} = \frac{1}{187}\cdot\frac{wl_1^2}{12}(51 + 136m^2)$$

$$= \frac{1}{187}\cdot\frac{wl_2^2}{12}\left(\frac{51}{m^2} + 136\right) = \frac{2\cdot67m^2 + 1}{44m^2}w.l_2^2$$

and $$+M(\text{max.}) = \left(\frac{1}{8} - \frac{2\cdot67m^2 + 1}{44m^2}\right)wl_2^2 = \frac{2\cdot83m^2 - 1}{44m^2}.wl_2^2$$

(b) If $m \geq 1$, $w_1 = w_3 = w$ and $w_2 = 0$.

Therefore $$M_{BG} = -M_{GB} = -\frac{1}{187}\cdot\frac{wl_1^2}{12}(68 + 119m^2) = -\frac{1\cdot75m^2 + 1}{33m^2}.wl_2^2$$

and $$+M(\text{max.}) = \left(\frac{1}{8} - \frac{1\cdot75m^2 + 1}{33m^2}\right)wl_2^2 = \frac{2\cdot38m^2 - 1}{33m^2}.wl_2^2$$

The conditions for maximum bending moments at the corners for cases (a) and (b) respectively are the same as for the maximum moments in the span in cases (b) and (a) respectively.

EXAMPLE 2.—Derive formulæ for the maximum bending moments at the supports of continuous beams with three unequal spans supported by unyielding supports, each span having a different moment of inertia. The load is $w = g + p$ where g is the dead load and p is the live load. Assume a different load, w_1, w_2, and w_3, for each span (*Fig.* 2). From Clapeyron's equations,

$$2M_1\left(\frac{l_1}{I_1} + \frac{l_2}{I_2}\right) + M_2\frac{l_2}{I_2} = -\frac{w_1l_1^3}{4I_1} - \frac{w_2l_2^3}{4I_2};$$

$$M_1\frac{l_2}{I_2} + 2M_2\left(\frac{l_2}{I_2} + \frac{l_3}{I_3}\right) = -\frac{w_2l_2^3}{4I_2} - \frac{w_3l_3^3}{4I_3}$$

Writing $\frac{I_1}{l_1} = K_1$, $\frac{I_2}{l_2} = K_2$, and $\frac{I_3}{l_3} = K_3$ the solution is

$$M_1 = -\frac{K_1}{K_1(4K_2 + 3K_3) + 4K_2(K_2 + K_3)}\left[\frac{K_2(K_2 + K_3)}{2K_1}.w_1l_1^2 + \frac{2K_2 + K_3}{4}.w_2l_2^2 - \frac{K_2}{4}w_3l_3^2\right]$$

$$M_2 = -\frac{K_3}{K_1(4K_2 + 3K_3) + 4K_2(K_2 + K_3)}\left[-\frac{K_2}{4}.w_1l_1^2 + \frac{K_1 + 2K_2}{4}.w_2l_2^2 + \frac{K_2(K_1 + K_2)}{2K_3}.w_3l_3^2\right]$$

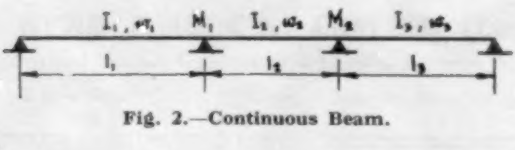


Fig. 2.—Continuous Beam.

With the usual assumption that $I_1 = I_2 = I_3 = I$, that is $K_1 = \frac{I}{l_1}$, $K_2 = \frac{I}{l_2}$, and $K_3 = \frac{I}{l_3}$, and relating all bending moments to the internal span l_2 by writing $\frac{l_1}{l_2} = m$ and $\frac{l_3}{l_2} = n$,

$$M_1 = -\frac{n}{3 + 4(m + n + mn)}\left[\frac{m^3}{2}.\frac{n+1}{n}.w_1 + \frac{2n+1}{4n}.w_2 - \frac{n^2}{4}.w_3\right].l_2^2$$

$$M_2 = -\frac{m}{3 + 4(m + n + mn)}\left[-\frac{m^2}{4}.w_1 + \frac{2m+1}{4m}.w_2 + \frac{n^3}{2}.\frac{m+1}{m}.w_3\right].l_2^2$$

The decisive load for the maximum bending moments within the first and third spans is $w_1 = w_3 = w$ and $w_2 = g$, hence

$$M_1' = -\frac{n}{3 + 4(m + n + mn)}\left[\left(\frac{m^3}{2}.\frac{n+1}{n} - \frac{n^2}{4}\right)w + \frac{2n+1}{4n}.g\right].l_2^2$$

$$M_2' = -\frac{m}{3 + 4(m + n + mn)}\left[\left(-\frac{m^2}{4} + \frac{n^3}{2}.\frac{m+1}{m}\right)w + \frac{2m+1}{4m}.g\right]l_2^2$$

The decisive load for the maximum bending moment within the second span is $w_1 = w_3 = g$ and $w_2 = w$. Hence

$$M_1'' = -\frac{n}{3 + 4(m + n + mn)}\left[\left(\frac{m^3}{2}.\frac{n+1}{n} - \frac{n^2}{4}\right)g + \frac{2n+1}{4n}.w\right].l_2^2$$

$$M_2'' = -\frac{m}{3 + 4(m + n + mn)}\left[\left(-\frac{m^2}{4} + \frac{n^3}{2}.\frac{m+1}{m}\right)g + \frac{2m+1}{4m}.w\right].l_2^2$$

The maximum bending moments at the supports for $w_1 = w_2 = w$ and $w_3 = g$ are

$$M_1(\text{max.}) = -\frac{n}{3 + 4(m + n + mn)}\left[\left(\frac{m^3}{2}.\frac{n+1}{n} + \frac{2n+1}{4n}\right)w - \frac{n^2}{4}g\right].l_2^2$$

For $w_1 = g$ and $w_2 = w_3 = w$,

$$M_2(\text{max.}) = -\frac{m}{3 + 4(m + n + mn)}\left[-\frac{m^2}{4}g + \left(\frac{2m+1}{4m} + \frac{n^3}{2}.\frac{m+1}{m}\right)w\right].l_2^2$$

Evidently, if $m = n = 1$, that is the three spans are equal,

$$M_1' = M_2' = M_1'' = M_2'' = -\left(g + \frac{p}{2}\right)\frac{l^2}{10}$$

and

$$M_1(\text{max.}) = M_2(\text{max.}) = -\left(\frac{g}{10} + \frac{7p}{60}\right)l^2.$$

Experimental Road in Nottinghamshire.

A NEW section of the experimental concrete road known as the Oxton by-pass, which connects the Leicester to Newark road (A46) and the Nottingham to Bawtry road (A614), has been completed. Part of the by-pass was built before the war. In a further section, begun in 1946, experiments were made with reinforced and unreinforced slabs of various thicknesses. In 1955 work was started on the southern half of the by-pass, and methods were used that were considered to be suitable for much larger works and also for improving the riding qualities of concrete roads and gaining experience in applying to roads the methods used for the rapid construction of airfield runways. A high standard of riding quality was set for this road with an irregularity index of not more than 40 in. per mile. The carriageway is 8 in. thick, 24 ft. wide, and 1300 yd. long. The concrete was made with ordinary Portland cement mixed in the proportions of 1 : 6·25 and had a water-cement ratio varying from 0·55 to 0·58. The coarse aggregate was in two sizes and was mixed with sand in the proportions of 2 parts sand, 1·6 parts $\frac{3}{8}$ in. aggregate and 3 parts $\frac{3}{4}$ in. aggregate. These proportions were varied slightly during the course of the work. Test cores had an average strength equivalent to cubes with a 28-day strength of 7000 lb. per square inch.

Fig. 1.—Supplying Concrete to Hopper of Machine.

The arrangement of the plant is shown in *Fig.* 2. The cement and aggregate were weighed and batched at a central plant and taken in a dry state to the mixer behind the road-making machine, where they were mixed at a rate of 60 cu. yd. per hour. Water was delivered in "tanker" road vehicles. The concrete was spread by a new type of machine designed by the contractors, Messrs. John Laing & Son, Ltd. The concrete was

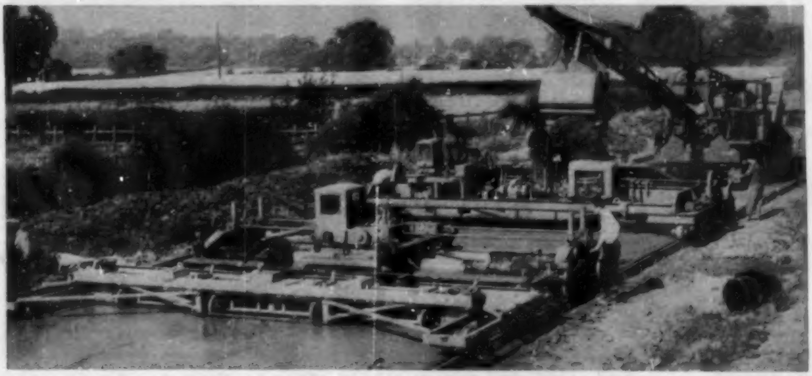


Fig. 2.—The Road-making Machine.

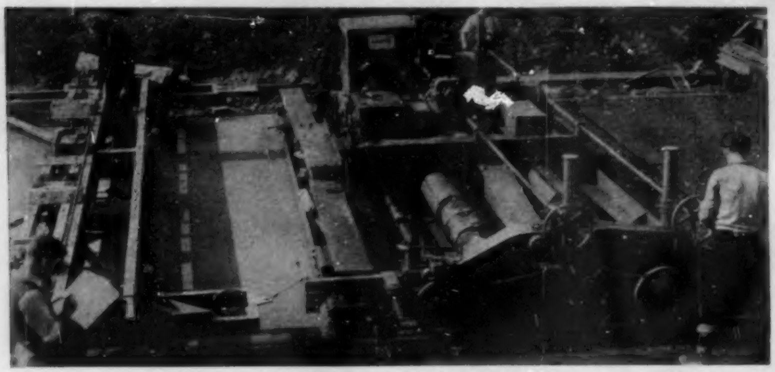


Fig. 3.—The Tamping and Finishing Beams.

emptied from the mixer into a bottom-opening bucket, which was moved along a beam and emptied into a hopper on the machine spanning the full width of the slab (*Fig.* 1). Below the hopper were two screws, one left-handed and the other right-handed, which distributed the concrete across the road. The concrete was levelled as the machine moved forward.

The spreader was followed by a vibrating compacting beam and an oscillating smoothing beam (*Fig.* 3). This was followed by a profilometer, which recorded any deviation from the required surface. Such deviations were corrected by returning the beams to repeat the operation. The spreader, finishing beams, and profilometer were mounted on rails fixed to the steel forms along the edge of the slab. These forms were 10 ft. long and 8 in. deep, and were laid on a previously-prepared concrete foundation.

A mesh of reinforcement weighing about 10 lb. per square yard was placed about 2½ in. below the surface of the slab. Instead of supporting the reinforcement on stirrups as is customary, it was supported on a skid which was moved forward by the spreader. After the machine had moved on, the surface of the concrete was brushed to increase its resistance to skidding and to diminish glare from the sun or headlights, and was then sprayed with a curing liquid.

Expansion joints are at intervals of 90 ft. to 120 ft. The joint was made by a wooden board ¾ in. thick and 6¼ in. deep placed across the width of the road

Fig. 4.—An Expansion Joint.

Fig. 5.—Cutting a Joint.

G*

(*Fig.* 4). Dowel bars 2 ft. long, with a cardboard cap at one end to allow for expansion, were placed in position in the board. A triangular fillet was placed on the top of the board with its apex ⅜ in. below the surface of the concrete, so as to ensure that shrinkage cracks would be confined to this line. The machine moved across the joint, placing concrete without interruption.

When the concrete had hardened a diamond saw was used to cut through the concrete above the wooden joint-filler (*Fig.* 5) and the fillet was removed. The groove so formed was then cleaned, primed and filled with a sealing material. A dummy joint was cut longitudinally along the centre of the road to prevent any irregular cracking across the centre of the slab.

Britain's First Motor Road Started.

WORK on a new road exclusively for the use of motor vehicles, and the first of its kind in this country, started in June. The new road, known as the Preston by-pass, will be more than eight miles long and will be the first part of the proposed north to south motorway from Birmingham to north of Shap. The section now started will extend from near the junction of the A6 and A49 roads at Bamber Bridge to the Preston–Lancaster road south of Broughton.

The new road will have dual carriageways each 24 ft. wide with space to widen them to 36 ft. There will be no kerbs, but the sides of the road will be marked by a coloured strip beyond which will be a parking strip 8 ft. wide. Side roads and footpaths will pass under or over the road. There will be no footpaths. The work includes two welded-steel bridges. It is estimated that the work will cost £3,000,000 and will require about two years to complete. The scheme was prepared by the Surveyor and Bridgemaster of the Lancashire County Council.

Centering for Ribbed Floors.

THE following notes describe the hollow concrete floors at a hospital recently completed at Limerick, Eire, at a cost of about £1,000,000. The floor and roof slabs generally are of "hollow steel mould" construction, having spans up to 26 ft. and overall thicknesses of floors varying from 12 in. to 15 in. These floors have a total area about 180,000 sq. ft., and are designed for imposed loads of 30 lb. to 100 lb. per square foot in addition to finishes and partitions.

The floors (*Fig.* 1) consist of thin reinforced concrete ribs at about 2 ft. centres with a 2-in. reinforced concrete slab on top to form tee-beams and to provide the floor surface. The trough-shaped steel moulds which form the centering are in standard lengths of 2 ft. and 3 ft., and these sizes, with cover moulds 1 ft. 2 in. long, are used for any required span. The moulds are fixed by wedging them into channel-shaped shoes previously fixed on the staging. *Fig.* 2 shows a method of setting out the shoes on timber bearers, but special couplers are available if the whole of the staging is of tubular steel. The moulds are in depths of 9 in. and 15 in., and the ribs may be in depths of 4 in. to 15 in. by the use of timber blocking pieces. Spans may be up to 31 ft. in reinforced concrete and up to 45 ft. in prestressed concrete. *Fig.* 3 shows the centering in position.

The shoes shown in the illustration were 5 in. wide, but widths of 3 in. and 3½ in. are also supplied. As the moulds are of standard width, the spacing of the ribs varies with the dimension of the shoe between 1 ft. 11½ in. and 2 ft. 2 in. Where the ribs meet beams or walls, steel end-pieces are used to form side shutters between the ribs. Alternatively these stop-ends may be made of timber on the site.

In the main blocks of this hospital the wards have balconies on to which the beds can be moved; these are cantilevered parts of the floor slabs, and some of the moulds were omitted to permit the floors to be solid where they are subjected to compression due to the cantilever. The partitions generally are carried on the ribbed floors, but where heavy partitions and local loads occur the moulds were spaced farther apart in order to form wider ribs. Construction joints were usually formed in the 2-in. topping, half way between ribs, by the use of strips of timber which also kept the mesh reinforcement in position and acted as screeds. Construction joints were formed at right-angles to the direction of the ribs and at the centre of the span.

The total weight of the centering in position, but excluding propping, is 75 lb. per square yard in the case of 15-in. moulds and 60 lb. per square yard in the

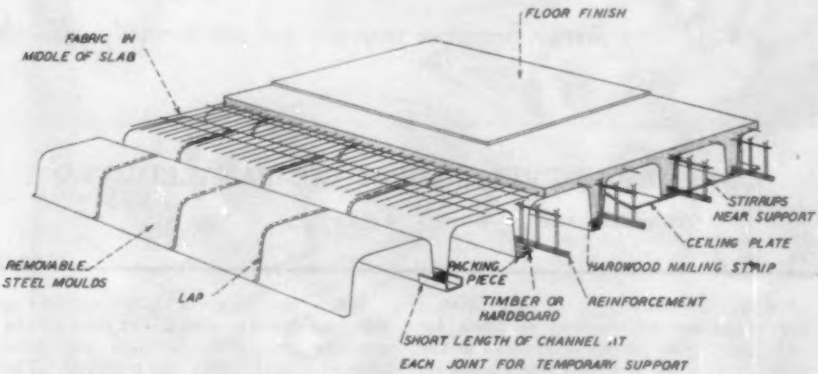


Fig. 1.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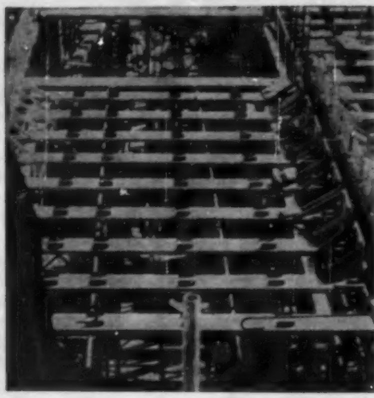

Fig. 2.

Fig. 3.

case of 9-in. moulds. The weight of this type of floor suitable for an imposed load of 80 lb. per square foot over a span of, say, 26 ft., is about 57 lb. per square foot.

Use was made of the spaces between the ribs for the installation of electrical services, hot and cold water supplies, waste pipes, and low-pressure hot-water heating. As far as possible the pipes and conduits are in the direction of the spans of the ribs, but where necessary pipe-sleeves were cast into the ribs to permit the passage of services at right angles to the span. Strips of cedarwood of splayed section were cast into the soffit of each rib (*Fig.* 1); to these, battens were fixed at 14-in. centres to carry plaster slabs finished with a skim coat of plaster. An alternative method of fixing the ceiling to the ribs is a permanent soffit lining piece 1 in. thick made of creosoted softwood and secured to the concrete by galvanised-wire ties. This generally provides a truer soffit line and a neater finish, but has the disadvantage that the maximum depth of rib possible is reduced to 8 in. for 9-in. moulds and to 14 in. for 15-in. moulds. Ceilings with metal hangers can be suspended from the ribs during casting, or the floor can be finished without a ceiling to expose the soffit of the ribs; this treatment was used in the boiler-house where the roof is 20 ft. above the floor. Where insulation is important, a roof of this type might be finished with sprayed asbestos instead of roughcast.

The architect for the Limerick Regional Hospital Board is Mr. Patrick J. Sheahan, K.S.S., F.R.I.A.I., M.I.C.E.I., who designed the building in consultation with Messrs. Stanley Hall, Easton and Robertson. The structural designs were prepared by the British Reinforced Concrete Engineering Co., Ltd., who are the proprietors of the "hollow steel mould" construction described. The general contractors were Messrs. Murphy Brothers (Cork), Ltd.

The Treatment of Concrete with Silicones.

It is reported in "Engineering News-Record" for November 24, 1955, that many concrete bridges on the New York State Thruway have been treated with a silicone solution to reduce the effect of freezing and thawing. The Authority states that silicone, by reversing the capillary attraction of concrete, reduces moisture absorption more effectively than any other surface treatment.

The specifications of New York State require air-entrained concrete to be used in bridge decks, but about 500 earlier bridges were built with a mixture of natural and Portland cements. Of these, 160 that are maintained by the Authority are being treated with silicone. Ultimately, this treatment will be given to decks built with air-entrained concrete.

The silicone treatment will replace the oil-spray treatment; where the silicone treatment has been applied to bridges that had been treated with oil the water repellency of the surface was greatly increased. It is stated that silicone is hydrophobic, rapidly dries out without any tendency to attract air-borne dust and dirt, and its resistance to oxidation indicates that it retains its ability to repel water for longer than the best organic oils.

A spraying equipment for treating the bridges consists of a tank of 230 gallons capacity mounted on a truck together with a rotary pump driven by a 5-h.p. petrol engine. This pump supplies a 14-ft. spray bar made up of a 6-ft. section and two independently controlled 4-ft. sections to provide flexibility. There are also two hand spray-bars for work that cannot be reached by the main spray-bars. The truck that carries the tank and spray has an operating speed of about three miles an hour. One gallon of 2 per cent. silicone solution covers about 100 sq. ft. and the cost is about 32*s*. per gallon of 20 per cent. solution. The bridge decks are swept with rotary brushes in advance of the treatment, to ensure that dust does not absorb much of the solution.

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Printed by Butler & Tanner Ltd., The Selwood Printing Works, Frome and London